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# Cherenkov Radiation: Electron Self-Energy Approach

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A short review is given of a previous paper on Cherenkov radiation (CR) by Fülöp and Biró (1992), together with some aspects of the collective dipole oscillations of atomic microclusters (atomic analog of the nuclear giant resonance). Finally, the CR energy loss of a relativistic charged particle in matter is obtained by evaluating the self-energy of the particle in the medium.

The energy loss of a relativistic charged particle in a polarizable medium has recently been studied by Fülöp and Biró (1992) by evaluating the work done by the current on the electric field it generates

$$\frac{dW}{dt} = -\int_{-\infty}^{\infty} \mathbf{j}(\mathbf{r}, t) \cdot \mathbf{E}(\mathbf{r}, t)\, d^3r = -e\mathbf{v} \cdot \mathbf{E}(\mathbf{v}t, t) \tag{1}$$

leading to the energy loss expression

$$\frac{dW}{dx} = \frac{ie^2}{\pi} \int\int_{-\infty}^{\infty} \frac{[1/\varepsilon(\omega)v^2 - 1/c^2]\omega q}{q^2 + \omega^2[1/v^2 - \varepsilon(\omega)/c^2]}\, dq\, d\omega \tag{2}$$

From this equation we obtained the CR energy (Fülöp and Biró, 1992)

$$\frac{dW}{dx} = \frac{e^2}{c^2} \int \left(1 - \frac{1}{\beta^2 \operatorname{Re} \varepsilon(\omega)}\right) \omega\, d\omega \tag{3}$$

where

$$\beta^2 \operatorname{Re} \epsilon(\omega) > 1 \tag{4}$$

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$\varepsilon(\omega)$ is the dielectric function at the point $\mathbf{r} = \mathbf{v}t$ occupied by the charge

$$\varepsilon(\omega) = 1 + \frac{a_0}{\omega_0^2 - \omega^2 - ig\omega}$$

The polarization state moves with the particle, and the distorted atoms (electric dipoles) in the neighborhood of the particle oscillate with the frequency $(\omega_0^2 - g^2/4)^{1/2}$ and lifetime $(g/2)^{-1}$. The damping constant $g$ has the magnitude of $\omega_0$. A part of the energy absorbed by the atoms is remitted; the constructive interference of the emitted radiation is interpreted as CR. In the frequency range for which (4) holds ($\beta > \beta_{\text{thr}}$) the integral is finite, and the inequality $\beta n(\omega) > 1$ follows from (4). $n(\omega)$ is the refractive index of the transparent medium. The radiation with frequency $\omega$ is observed at an angle $\theta(\omega) = \arccos[1/\beta n(\omega)]$ relative to the particle track. Having evaluated the field of a charged particle with constant velocity and taking into account the self-interaction character of equation (1), the electromagnetic origin of the energy loss is suggested, to which we turn later.

At this point let us remark on some aspects of the theoretical description of the collective dipole oscillations in metal microclusters (de Heer *et al.*, 1987). The photoabsorption cross section of small neutral sodium clusters composed of $n = 2$–$40$ sodium atoms irradiated with visible light has a frequency dependence

$$\sigma(\omega) = 4\pi \frac{ne^2}{m_e c} \frac{\omega^2 \Gamma}{(\omega_0^2 - \omega^2)^2 + \omega^2 \Gamma^2}$$

Here the damping constant $\Gamma$ is the only free parameter which has to be adjusted from the measured cross-section data. The best fit with the measured cross section is found by setting $\Gamma/\omega_0 = 0.15$ (de Heer *et al.*, 1987). The analogy between the collective vibrations of the valence electrons in small metal clusters and the vibrations of protons against neutrons (giant dipole resistance) in nuclei is reviewed by Broglia *et al.* (1991). We believe that the damping mechanism of CR we proposed is closely related to these collective resonance effects rather than to the physics of the radiation emitted by an accelerated charged particle or charged particle bunches. Recently, Shibata *et al.* (1991) communicated the observation of CR at millimeter wavelengths emitted from bunch electrons. The number of electrons in a bunch is estimated to be $2.65 \times 10^{10}$, the medium is nondispersive ($n = 1.00027$), no angle of emission is reported, and the spectrum is similar to the synchrotron radiation spectrum (Shibata *et al.*, 1991). This kind of radiation we do not call CR.

A similar comment holds on the nomenclature of CR X-rays (Caticha, 1992), which have various other names, such as parametric X-ray, or dynamic radiation, and which are more appropriate for the phenomenon considered in our opinion.

Although in the classical treatment of CR (Landau and Lifshitz, 1984) it is emphasized that this radiation is completely different from the bremsstrahlung, the CR is emitted by the medium under the action of the electric field of the particle, in quantum field theory approaches (Harris, 1972; Ahlen, 1980) the CR is considered to be a first-order process in which the radiation is emitted by the particle by changing its momentum $\Delta\mathbf{p}$ with $\hbar\mathbf{k}$ (Harris, 1972; Ahlen, 1980). It is to be noted, however, that in such a process (represented in Fig. 1) the four-momentum cannot be conserved for real physical particles ($k^2=0$ for photons and $p^2=m_0^2c^2$ for electrons); generally speaking, no real first-order process can occur (Schweber, 1962; Mandl and Shaw, 1984).

Returning to the collective effects discussed previously, of special interest is an article of Lindhard (1955) on the passage through matter of swift charged particles. Lindhard pointed out that in a gas of free electrons, oscillations can occur with the frequency $\Omega^2=4\pi Ne^2/m$; the oscillations interfere destructively with the force from the particle when the collision time is longer than the oscillation time $\Omega^{-1}$. The energy loss expression is similar to (2), and given this equation, which is directly connected with the imaginary part of the energy of the particle, one can find its self-energy in the medium (Lindhard, 1955). In the remaining part of our paper we indicate a way of getting equation (2) starting from the electron self-energy diagram (Fig. 2), following with some modifications Crispin and Fowler (1970) and Ter-Mikaelian (1972).

The self-energy or mass operator of the incident charged particle may be written as

$$\Sigma_1^{(2)}=\frac{2\alpha}{(2\pi)^4}\frac{i\hbar^2}{c}\int_{-\infty}^{\infty}d\omega\int_{-\infty}^{\infty}d^3k\left[\frac{4\pi(\mathbf{v}\times\mathbf{k})^2}{(\mathbf{k}^2-\omega^2\varepsilon(\omega)/c^2)\mathbf{k}^2}-\frac{4\pi c^2}{\mathbf{k}^2\varepsilon(\omega)}\right]$$
$$\times(E-E'-\hbar\omega+i\delta)^{-1}\equiv\int_{-\infty}^{\infty}\Sigma_1\omega)\,d\omega \tag{3}$$

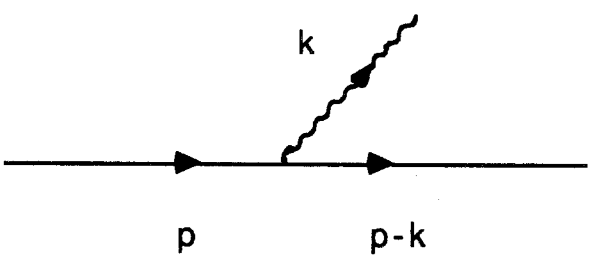


**Fig. 1.** Unphysical bremsstrahlung.

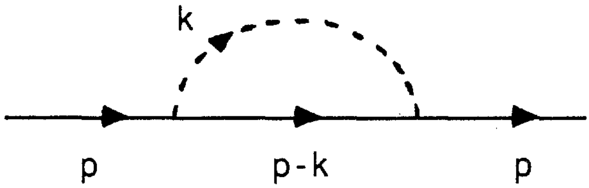


**Fig. 2.** The electron self-energy.

where the photon propagator is separated into a longitudinal and a transverse part, $\varepsilon(\omega)$ is the dielectric function describing the polarization production by the virtual photon flux, $\alpha = e^2/\hbar c$, and $E' = c|\mathbf{p} - \hbar \mathbf{k}|$ in the extreme relativistic approximation. In this case $E - E' - \hbar\omega = -\hbar \mathbf{k} \cdot \mathbf{v} + \hbar\omega$, which enables us to replace the particle propagator according to

$$(E - E' - \hbar\omega + i\delta)^{-1} \rightarrow -i\pi\delta(\hbar w - \hbar \mathbf{k} \cdot \mathbf{v}) \tag{6}$$

It should be noted that the condition $\omega = \mathbf{k} \cdot \mathbf{v}$ is an exact relation if the condition of transversality of the current is imposed, $k_\mu j^\mu = 0$, with $j_\mu = (ec, e\mathbf{v})$ and $k_\mu = (\omega/c, k)$.

Substitution of (6) into (5) gives

$$\tilde{\Sigma}_1^{(2)} = \frac{\alpha v \hbar}{2\pi c}\left[\int_{-\infty}^{\infty} d\omega \int_0^\infty \frac{q^2 d(q^2)}{q^2 + \omega^2/v^2)(q^2 + \tilde{z})} - \frac{c^2}{v^2}\int_0^\infty \frac{d(q^2)}{(q^2 + \omega^2/v^2)\varepsilon(\omega)}\right]$$

where cylindrical polar coordinates were introduced $[q^2 + k_y^2 + k_z^2,\ dk_y.\ dk_z = \pi d(q^2),\ k_x = \omega/v]$ and

$$\tilde{z} = \frac{\omega^2}{v^2}\left[1 - \frac{v^2}{c^2}\varepsilon(\omega)\right]$$

A little algebra leads to

$$\tilde{\Sigma}_1^{(2)} = \frac{\alpha v \hbar}{2\pi c}\int_{-\infty}^{\infty} d\omega \int_0^\infty d(q^2)\frac{v^2\varepsilon(\omega) - c^2}{v^2\varepsilon(\omega)}\frac{1}{q^2 + \tilde{z}} \equiv \int_{-\infty}^{\infty} \tilde{\Sigma}_1(\omega)\, d\omega$$

The energy loss is connected to the imaginary part of $\tilde{\Sigma}_1(\omega)$ by $W \sim (1/v) \int_0^\infty \omega \operatorname{Im} \tilde{\Sigma}_1(\omega)\, d\omega$, or, following Ter-Mikaelian (1972) in writing the particle energy as $\Delta\tilde{E} = \Delta E - i\gamma\hbar/2$ with the attenuation

$$\gamma = 2i\frac{e^2 v}{\pi\hbar} \operatorname{Im} \int_0^\infty d\omega \int_0^\infty \frac{q(1/v^2 - \varepsilon(\omega)/c^2)\, dq}{[q^2 + \omega^2(1/v^2 - \varepsilon(\omega)/c^2)]\varepsilon(\omega)}$$

the energy loss expression is

$$\frac{dW}{dx} = i\frac{e^2}{\pi}\int_{-\infty}^{\infty} \omega\, d\omega \int_0^\infty \frac{q(1/v^2 - \varepsilon(\omega)/c^2)\, dq}{[q^2 + \omega^2(1/v^2 - \varepsilon(\omega/c^2)]\varepsilon(\omega)}$$

which is exactly (2). From this we get the CR energy loss, equation (3), and the ionization plateau is obtained by introducing a cutoff $q_0$ (Landau and Lifshitz, 1984)

$$\frac{dW}{dx} = \frac{2\pi Ne^4}{mc^2} \ln \frac{mc^2 q_0^2}{4\pi Ne^2} \tag{7}$$

In Fermi's (1940) original formula (equation (33)] the cutoff is the inverse of a minimum distance from the particle's track, and it emphasized that in the limiting case $v \to c$ the ionization plateau is independent of both the binding frequency and the damping constant. The common use of $\varepsilon(\omega)$ in the ionization and CR process for very high energy is therefore legitimate. Another point of view of the dielectric function and the polarization energy $\Delta E$ (without reference to CR) leads to the mass renormalization equation $\Delta m_{\text{vac}} - \Delta m_{\text{med}} = e^2\omega_0/c^3$ (Ter-Mikaelian, 1972).

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